



Christ Church  
Grammar School

2016  
UNIT TEST 2

## MATHEMATICS SPECIALIST Year 12

Section One:  
Calculator-free

Student name \_\_\_\_\_

Teacher name \_\_\_\_\_

### Time and marks available for this section

Reading time before commencing work: 2 minutes  
Working time for this section: 15 minutes  
Marks available: 15 marks

### Materials required/recommended for this section

*To be provided by the supervisor*

This Question/Answer Booklet  
Formula Sheet

*To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Instructions to candidates**

1. Write your answers in this Question/Answer Booklet.
2. Answer all questions.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that **you do not use pencil**, except in diagrams.

## Question 1

(5 marks)

Consider the polynomial  $p(x) = x^3 + ax^2 + bx + 1$ , where  $a$  and  $b$  are **real** numbers.

The polynomial  $p(x)$  has **the same** remainder when divided by  $(x + 2)$  as when divided by  $(x - i)$ , where  $i = \sqrt{-1}$ .

Determine the values of  $a$  and  $b$ .

(Hint: The remainder theorem is also true for complex numbers!)

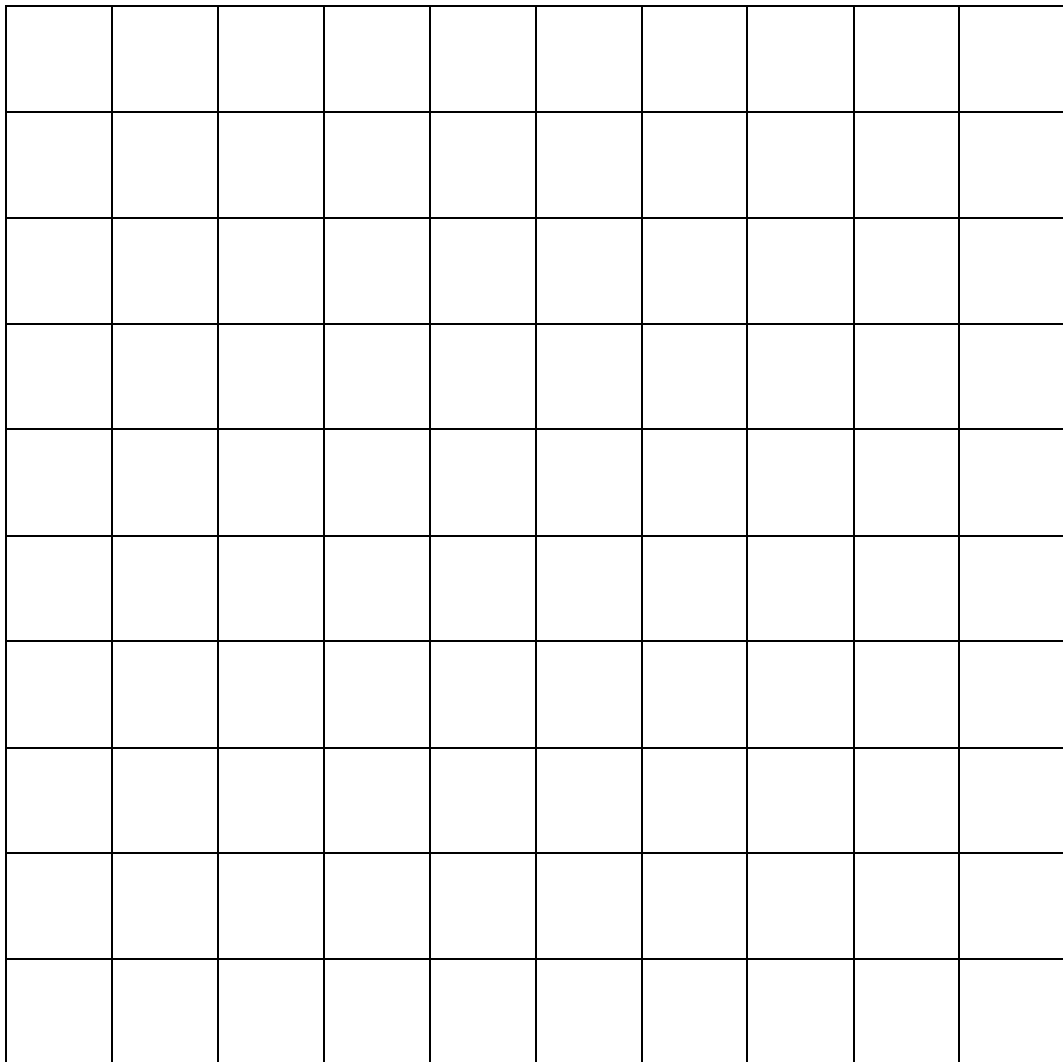
## Question 2

(10 marks)

Let  $f(x) = \sqrt{4x - 3}$ .

- (a) State the domain of  $f(x)$ . (1 mark)
- (b) Find an expression for the inverse function  $f^{-1}(x)$ . (2 marks)
- (c) Use algebra to determine the coordinates of the points where the graphs of  $y = f(x)$  and  $y = f^{-1}(x)$  intersect. (3 marks)

- (d) On the same set of axes, sketch the graphs of  $y = f(x)$  and  $y = f^{-1}(x)$ , labelling all axis-intercepts and intersection points with their coordinates. (4 marks)





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## MATHEMATICS SPECIALIST Year 12

Section Two:

Calculator-assumed

Student name \_\_\_\_\_

Teacher name \_\_\_\_\_

### Time and marks available for this section

Reading time before commencing work: 3 minutes  
Working time for this section: 30 minutes  
Marks available: 30 marks

### Materials required/recommended for this section

*To be provided by the supervisor*

This Question/Answer Booklet  
Formula Sheet (retained from Section One)

*To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

### Important note to candidates

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**Instructions to candidates**

1. Write your answers in this Question/Answer Booklet.
2. Answer all questions.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that **you do not use pencil**, except in diagrams.

**Question 3****(5 marks)**

Recall from Year 10 that you can use *Polynomial Long Division* to divide a polynomial  $P(x)$  by a divisor  $D(x)$ , resulting in a quotient  $Q(x)$  and a remainder  $R(x)$ . You probably wrote this as

$$P(x) = D(x)Q(x) + R(x)$$

You may also recall the *Remainder Theorem*:

When a polynomial  $P(x)$  is divided by  $(x - a)$ , the remainder is  $P(a)$ .

You may even recall The *Factor Theorem*:

For a polynomial  $P(x)$ , if  $P(a) = 0$ , then  $(x - a)$  is a factor of  $P(x)$ .

(a) Prove the *Remainder Theorem*.

**(3 marks)**

(b) Prove the *Factor Theorem*.

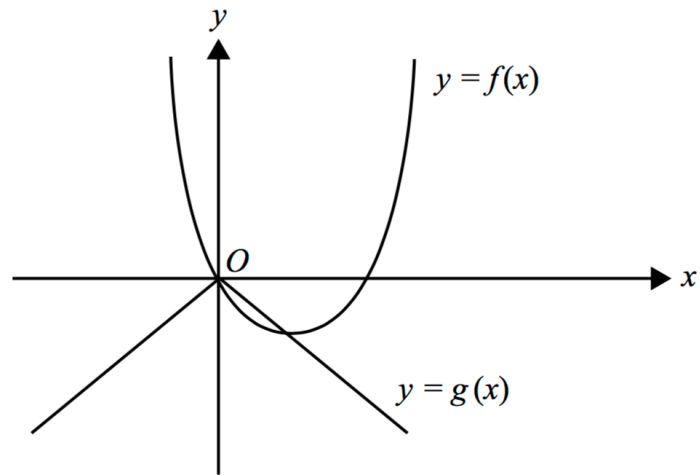
**(2 marks)**



Question 4

(5 marks)

The graphs of  $y = f(x)$  and  $y = g(x)$  are as shown below.



Sketch the graph of  $y = f(g(x))$ .

## Question 5

(7 marks)

Let  $f(x) = x^3 - x^2 + 3x - 3$ .

(a) Show that  $(x - 1)$  is a factor of  $f(x)$ . (2 marks)

(b) Hence, or otherwise, find all the solutions to the equation  $f(x) = 0$ . (3 marks)

(c) In part (b) you would have noticed that two of the solutions (roots) were complex conjugates of each other. When will a cubic equation have two **non-real** solutions that are complex conjugates of each other? (2 marks)

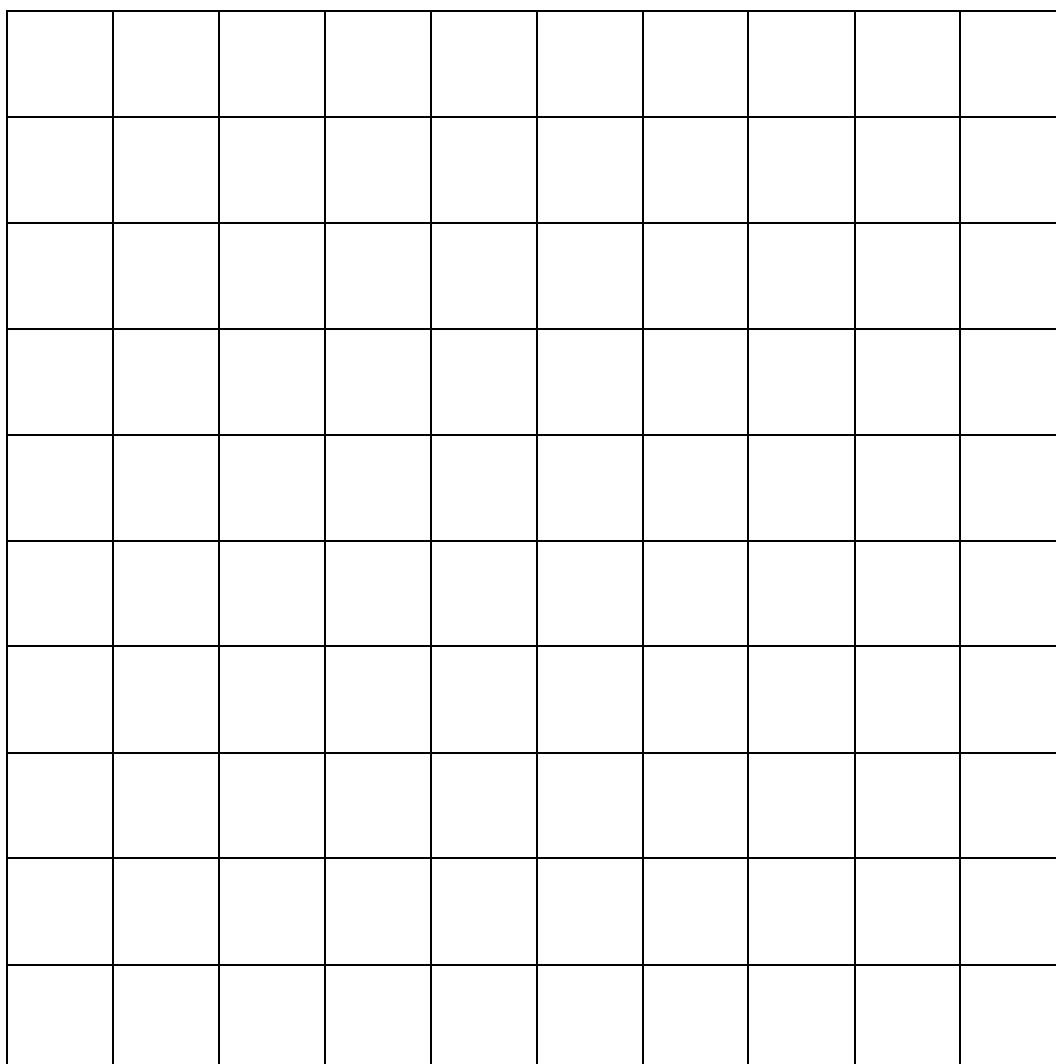
Question 6

(8 marks)

Let  $f(x) = x - \frac{1}{2}x^2$  for  $x \leq a$ .

(a) Determine the largest value of  $a$  such that  $f(x)$  has an inverse function. (2 marks)

(b) Sketch the graphs of  $y = f(x)$  and  $y = f^{-1}(x)$  on the same set of axes. (Use the same scale on both axes) (3 marks)



(c) Find an expression for  $f^{-1}(x)$ .

(3 marks)

## Question 7

(5 marks)

Consider the function

$$f(x) = \frac{x+1}{x-1}, x \neq 1.$$

(a) Show that  $f(f(x)) = x$ .

(2 marks)

(b) Use algebra to find an expression for the inverse function,  $f^{-1}(x)$ .

(3 marks)

**End of questions**